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Letter to the Editor

Constructal comment on a Fermat-type principle for heat flow

I want to bring to the readers' attention the connection between Professor Sieniutycz' "Fermat-type" principle [1] and the constructal principle, which was published four years earlier in the same journal [2]. This connection generalizes and strengthens the constructal theory [2–4] of how geometric form (optimized complexity, organization) is generated in macroscopic flow systems.

The constructal principle calls for maximizing global performance (objective, purpose, function) subject to global finiteness constraints, in a flow system the architecture of which is free to change. The result of invoking the constructal principle is the architecture of anything that moves—the very configuration (geometry, design, drawing) of the flow system.

The connection is summarized graphically in Fig. 1, based on the analogy between minimizing travel time [5], minimizing thermal resistance [2], and maximizing flow access in general [3,4]. This analogy is the theory. Constructal theory covers all flow systems; the animate, the inanimate and the engineered. The principle that generates optimal geometric form acts on both sides of Fig. 1. Fermat-type invocations of the principle (e.g. [1]) mean that the flow access is maximized between two discrete points. In the earliest papers on constructal theory [2–5], the flow access was maximized between one point (source, or sink) and an infinity of points (volume, or area). The principle works in considerably more complex systems, from power generation to bird flight [4].

For example, on the right side of Fig. 1 the fixed rectangular area H_0L_0 can have its shape H_0/L_0 optimized so that the travel time from the entire area (points of type Q) to the single point M is minimized. When the rectangle has the optimal shape, the travel at low speed V_0 is "balanced" against the travel at higher speeds (e.g., V_1). Low speeds (high resistivities, high costs) occupy the white area, while high speeds (low resistivities, low costs) characterize flow concentrations located on the black centerline (streets, ducts, business links). The macroscopically visible geometry represents the optimal allocation of imperfection—the allocation of high-resistance flow areas to low-resistance lines. Constructal geometry is the optimal distribution of imperfection.

The conceptual result is the same on both sides for Fig. 1: flow geometry is being generated, i.e., it is being derived from principle. For the point–point flow, the generated geometry is a broken line (refracted ray) of a certain, unambiguous shape. For the point–volume flow, the geometry is a bundle of an infinite number of such broken lines, which on the right side of Fig. 1 covers fully (continuously) a rectangular block of a certain, unambiguous shape. These blocks can be compounded in a sequence of assembly and optimization steps (the constructal sequence) to cover larger volumes for point–volume flows [2–5]. In such constructs, the regions with faster flow (low resistance, e.g., V_1 in Fig. 1) form a tree, while the regions with slower flow (high resistance, V_0) form the interstitial spaces of the tree. Both parts of the tree drawing result from design, the links and the interstices.

In sum, the "dissipative structures" of irreversible thermodynamics (e.g., trees) can be derived from principle, in the same mental fashion that the refracted ray and many other natural architectures [3,4] are rationalized based on the same principle (e.g., round tubes, vascularization, bronchial trees, dendritic crystals, turbulence, eddies, Bénard convection, mud cracks, traffic routes, business, communication networks, etc.).

When the ratio of the propagation speeds (or thermal conductivities) of the two media is infinite, the predicted flow path has a 90° angle. This is the limit illustrated on the right side of Fig. 1 and in a 1996 paper [2]. When the ratio is finite, as in [1], each broken line has a certain, unambiguous angle that is less than 90°. This angle was also optimized in the earliest constructal papers [5–7] and books [3,4].

It seems to me that what flows (light, heat, fluid, goods, electric power, people) is not nearly as important (general, powerful) as the principle that governs the generation of flow organization in all cases. A principle that works so well, from Fermat and Darwin to constructal theory today, deserves to be recognized as law. The constructal-law statement of 1996 covers all these observations: "For a finite-size system to persist in time, it must evolve in such a way that it provides easier access to the imposed currents that flow through it." [2]. This law brings into physics the concepts of objective, constraints and structure, i.e., optimization and performance. The constructal law is distinct from the second law (cf. [4], pp. xviii–xix). The second law is about the

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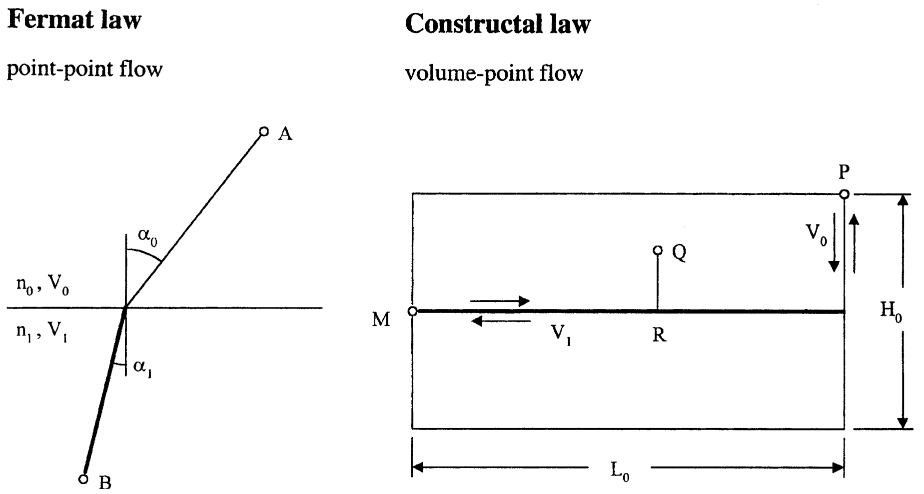

Fig. 1. The generation of geometric form through the minimization of travel time: travel between two points (Fermat principle), as a simple case of the constructal principle: travel between one point and an infinity of points (volume) [4].

generation of entropy. The constructal law is about the generation of geometry. Further discussions of constructal theory are provided in Refs. [8–11].

References

[1] S. Sieniutycz, Dynamic programming approach to a Fermat-type principle for heat flow, Int. J. Heat Mass Transfer 43 (2000) 3453–3468.
[2] A. Bejan, Constructal-theory network of conducting paths for cooling a heat generating volume, Int. J. Heat Mass Transfer 40 (1997) 799–816, Note: this issue was published on 1 November 1996.
[3] A. Bejan, Advanced Engineering Thermodynamics, second ed., Wiley, New York, 1997 (Chapter 13).
[4] A. Bejan, Shape and Structure, from Engineering to Nature, Cambridge University Press, Cambridge, UK.
[5] A. Bejan, Street network theory of organization in nature, J. Adv. Transport. 30 (1996) 85–107.
[6] G.A. Ledezma, A. Bejan, M.R. Errera, Constructal tree networks for heat transfer, J. Appl. Phys. 82 (1997) 89–100.
[7] A. Bejan, G.A. Ledezma, Street tree networks and urban growth: optimal geometry for quickest access between a finite-size volume and one point, Physica A 255 (1998) 211–217.
[8] P. Bradshaw, Shape and structure, from engineering to nature, AIAA J. 39 (2001) 983.
[9] J. Cervantes de Gortari, Shape and structure, from engineering to nature, Int. J. Heat Mass Transfer 45 (2002) 1583.
[10] J. Riznic, Search for better performance has wide appeal, Engineering Dimensions July/Aug. 2001, pp. 40–41.
[11] A. Rossiter, Shape and structure, from engineering to nature, Chem. Eng. Progress (February, 2002) 81.

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